## Functions

## A function is a rule which operates on one number to give another number.

However, not every rule describes a valid function.
> We will examine whether given rule describes a valid function.
$>$ Get introduced to some of the mathematical terms associated with functions.
$>$ Practice and check our knowledge with regard to functions
The objective is to

- recognise when a rule describes a valid function,
- be able to plot the graph of a part of a function,
- find a suitable domain for a function, and find the corresponding range.


## What is a function?

A function is a rule which maps a number to another unique number. In other words, if we start off with an input, and we apply the function, we get an output. For example, we might have a function that added 3 to any number. So if we apply this function to the number 2, we get the number 5. If we apply this function to the number 8 , we get the number 11. If we apply this function to the number $x$, we get the number $x+3$. We can show this mathematically by writing $f(x)=x+3$. The number $x$ that we use for the input of the function is called the argument of the function. So if we choose an argument of 2 , we get $f(2)=2+3=5$. If we choose an argument of 8 , we get $f(8)=8+3=11$. If we choose an argument of -6 , we get $f(-6)=-6+3=-3$. If we choose an argument of $z$, we get If we choose an argument of $x^{2}$, we get $f(z)=z+3$. $F\left(x^{2}\right)$ $=x^{2}+3$.

It may seem that we can pick any number we choose for the argument. However, that is not the case, as we will see later. But because we do have some choice in the number we can pick, we call the argument the independent variable. The output of the function, e.g. $f(x), f(5)$, etc. Depends upon the argument, and so this is called the dependent variable.

## Plotting the graph of a function

If we have a function given by a formula, we can try to plot its graph. Suppose, for example, that we have a function $f$ defined by $f(x)=3$ Plotting the graph of a function If we have a function given by a formula, we can try to plot its graph. Suppose, for example, that we have a function $f$ defined by $f(x)=3 \times 2-4$. The argument of the function (the independent variable) is x , and the output (the dependent variable) is $3 \mathrm{x} 2-4$. So we can calculate the output of the function for different arguments: $f(0)=3 \times 02-4=-4 f(1)=3 \times 12-4=-1 f(2)=3 \times 22-4=8$ $\mathrm{f}(-1)=3 \times(-1) 2-4=-1 \mathrm{f}(-2)=3 \times(-2) 2-4=8$. We can put this information into a table to help us plot the graph of the function. $f(x) x f(x)-28-1-10-412-1884-1-212-4 x$ We can use the graph of the function to find the output corresponding to a given argument. For instance, if we have an argument of 2 , we start on the horizontal axis at the point where $\mathrm{x}=2$, and we follow the line up until we reach the graph. Then we follow the line across so that we
can read off the value of $f(x)$ on the vertical axis. In this case, the value of $f(x)$ is 8 . Of course we already know this, because $x=2$ is one of the values in our table. $f(x) 84-1-212-4-4$. The argument of the function (the independent variable) is x , and the output (the dependent variable) is $3 x^{2}-4$.

So we can calculate the output of the function for different arguments:
$f(0)=3 \times 0^{2}-4=-4$
$f(1)=3 \times 1^{2}-4=-1$
$f(2)=3 \times 2^{2}-4=8$
$f(-1)=3 \times(-1)^{2}-4=-1$
$f(-2)=3 \times(-2)^{2}-4=8$.
We can put this information into a table to help us plot the graph of the function.

| x | $\mathrm{f}(\mathrm{x})$ |
| :---: | :---: |
| -2 | 8 |
| -1 | -1 |
| 0 | -4 |
| 1 | 1 |
| 2 | 8 |


| $x$ | $f(x)$ |
| :---: | :---: |
| -2 | 8 |
| -1 | -1 |
| 0 | -4 |
| 1 | -1 |
| 2 | 8 |



We can use the graph of the function to find the output corresponding to a given argument. For instance, if we have an argument of 2, we start on the horizontal axis at the point where x $=2$, and we follow the line up until we reach the graph. Then we follow the line across so that we can read off the value of $f(x)$ on the vertical axis. In this case, the value of $f(x)$ is 8 . Of course we already know this, because $\mathrm{x}=2$ is one of the values in our table. $\mathrm{f}(\mathrm{x}) 8$.


